Eigen Values of an Interval Matrix

G. Veeramalai

Assistant Professor, Dept.of Mathematics, Annai Mathammal Sheela Engineering College, Erumapatty, Namakkal (Dist), Tamilnadu-637 013

Abstract

In this paper, we propose a new method to compute the Eigen values of an interval Matrix based on the modified interval arithmetic. We introduce the notions of determinant, characteristic equation and Eigen values of an interval matrix.

Keywords: *Interval arithmetic, Interval matrix, characteristic equation, Eigen values.*

1. INTRODUCTION

It is well known, that matrices play major role in various areas such as Mathematics, statistics, Physics, Engineering and technology, social sciences and many others. In real life, due to the inevitable measurement inaccuracy, we do not know the exact values of the measured quantities; we know, at best, the intervals of possible values. Consequently, we cannot successfully use traditional classical matrices and hence the use of interval matrices is more appropriate.

In this paper, we propose a new method for finding the Eigen values of an interval matrix which in turn helps us to solve system of linear interval equations in a better way[5].

Let $\tilde{a} = [a_1, a_2] = \{x : a_1 \le x \le a_2 ; x \in R\}$

If $\tilde{a} = a_1 = a_2 = a$,then $\tilde{a} = [a_1, a_2] = a$ is a real number (or a degenerate interval). We shall use the

terms *interval* and *interval number* interchangeably. We use IR to denote the set of all interval numbers on the real line R. The mid-point and width (or half-width) of an interval number $\tilde{a} = [a_1, a_2]$ are defined as $m(\tilde{a}) = \left(\frac{a_1 + a_2}{2}\right)$ and $w(\tilde{a}) = \left(\frac{a_2 - a_1}{2}\right)$.

1.1 INTERVAL ARITHMETIC

We recall a new type of arithmetic operations on interval numbers introduced in [2]: For $\tilde{x} = [x_1, x_2]$, $\tilde{y} = [y_1, y_2]$ and for $* \in \{+, -, \times, \div\}$

We define $\tilde{x} * \tilde{y} = [(m(\tilde{x}) * m(\tilde{y}) - k), (m(\tilde{x}) * m(\tilde{y}) + k)]$

Where $k = \min[(m(\tilde{x}) * m(\tilde{y}) - \alpha), (\beta - m(\tilde{x}) * m(\tilde{y}))]$

Where α and β are the end point of the interval $\tilde{x} \circ \tilde{y}$ under the existing interval arithmetic.

In particular,

i. **Addition**

$$\tilde{x} + \tilde{y} = [x_1 + x_2] + [y_1 + y_2]$$

$$= \{m(\tilde{x}) + m(\tilde{y}) - k,\ m(\tilde{x}) + m(\tilde{y}) + k\}$$

Where $k = \left\{ \dfrac{(y_2 + x_2) - (y_1 + x_1)}{2} \right\}$

ii. Subtraction

$$\tilde{x} - \tilde{y} = [x_1 + x_2] - [y_1 + y_2]$$

$$= \{m(\tilde{x}) - m(\tilde{y}) - k,\ m(\tilde{x}) + m(\tilde{y}) + k\} \text{Where } k = \left\{ \frac{(y_2 + x_2) - (y_1 + x_1)}{2} \right\}$$

iii. Multiplication

$$\tilde{x}.\tilde{y} = \tilde{x}\,\tilde{y} = [x_1 x_2][y_1 y_2]$$

$$= \{m(\tilde{x})m(\tilde{y}) - k,\ m(\tilde{x})m(\tilde{y}) + k\}$$

Where $k = \min[(m(\tilde{x})m(\tilde{y}) - \alpha), (\beta - m(\tilde{x})m(\tilde{y}))]$ Where $\alpha = \min(x_1y_1, x_1y_2, x_2y_1, x_2y_2)$ Where $\beta = \max(x_1y_1, x_1y_2, x_2y_1, x_2y_2)$.

iv. Inverse

$$\tilde{x}^{-1} = \frac{1}{\tilde{x}} = \frac{1}{[x_1, x_2]} = \left[\frac{1}{m(\tilde{x})} - k,\ \frac{1}{m(\tilde{x})} + k \right]$$

Where $k = \min\left\{ \dfrac{1}{x_2}\left(\dfrac{x_2 - x_1}{x_1 + x_2}\right), \dfrac{1}{x_1}\left(\dfrac{x_2 - x_1}{x_1 + x_2}\right), \right\}$ &

$$0 \notin [x_1, x_2]$$

From iii. It is clear that $\lambda\tilde{x} = \begin{cases} [\lambda x_1, \lambda x_2] & for \lambda \geq 0 \\ [\lambda x_1, \lambda x_2] & for \lambda < 0 \end{cases}$

It is to be noted that we use $\circ$ to denote the existing interval arithmetic and $*$ to denote the modified interval arithmetic. But wherever there is no confusion we use the same notation for both the cases.

It is also to be noted that $\tilde{x} \circ \tilde{y} \subseteq \tilde{x} * \tilde{y} \left\{ x \circ y \middle/ x \subseteq \tilde{x}, y \subseteq \tilde{y} \right\}$, where $\circ \in \{\otimes, \oplus, \Theta, \mathrm{O}\}$ is the existing interval arithmetic.

For example: If $\tilde{x} = [-1,2]$ and $\tilde{y} = [3,5]$, then $\tilde{x} \otimes \tilde{y} = [-1,2] \otimes [3,5]$

$$= [\min(-3,-5,6,10\ ),\ \max(-3,-5,6,10)]$$
$$= [-5,10]$$

and $\tilde{x}.\tilde{y} = \tilde{x}\tilde{y} = [-1,2][3,5] = [-5,9]$ So that $\tilde{x} * \tilde{y} \subseteq \tilde{x} \circ \tilde{y}$

2. PRELIMINARY NOTES

2.1 DEFINITION OF INTERVAL MATRIX

An interval matrix $\tilde{A}$ is a matrix whose elements are interval numbers. An interval matrix $\tilde{A}$ will be written as

$$\tilde{A} = \begin{bmatrix} \tilde{a}_{11}\ \tilde{a}_{12}\ \ ..\ ..\tilde{a}_{1n} \\ \tilde{a}_{21}\ \tilde{a}_{22}\ ...\ \tilde{a}_{2n} \\ . \\ . \\ \tilde{a}_{m1}\ \tilde{a}_{m2}\ \ ...\tilde{a}_{mn} \end{bmatrix} = (a_{ij})_{1\le i\le m, 1\le j\le n}$$

Where each $\tilde{a}_{ij} = [\underline{a}_{ij}, \overline{a}_{ij}]$ or $\tilde{A} = [\ \underline{A}\,\overline{A}]$ for some $\underline{A}, \overline{A}$ satisfying $\underline{A} \le \overline{A}$.

We use $D^{m\,X\,n}$ to denote the set of all $(m\,X\,n)$ interval matrixes.

2.2 THE MIDPOINT INTERVAL MATRIX

The midpoint (center) of an interval matrix $\tilde{A}$ is the matrix of midpoints of its interval elements defined as

$$m(\tilde{A}) = \begin{bmatrix} m(\tilde{a}_{11})\ m(\tilde{a}_{12})\ \ ..\ ..m(\tilde{a}_{1n}) \\ m(\tilde{a}_{21})\, m(\tilde{a}_{22})\ ...\ m(\tilde{a}_{2n}) \\ . \\ . \\ m(\tilde{a}_{m1})\, m(\tilde{a}_{m2})\ \ ...m(\tilde{a}_{mn}) \end{bmatrix}$$

The width of an interval matrix $\tilde{A}$ is the matrix of width of its interval elements defined as

$$w(\tilde{A}) = \begin{bmatrix} w(\tilde{a}_{11})\ w(\tilde{a}_{12})\ \ ..\ ..w(\tilde{a}_{1n}) \\ w(\tilde{a}_{21})\, w(\tilde{a}_{22})\ ....\ w(\tilde{a}_{2n}) \\ . \\ . \\ w(\tilde{a}_{m1})\, w(\tilde{a}_{m2})\ \ ...w(\tilde{a}_{mn}) \end{bmatrix}$$ which always non negative.

2.3 NULL INTERVAL MATRIX

We use 0 to denote the null matrix $\begin{bmatrix} 0 & 0 & .. \ldots 0 \\ 0 & 0 & \ldots\ldots 0 \\ . & & \\ . & & \\ 0 & 0 & \ldots\ldots 0 \end{bmatrix}$ and $\tilde{0}$ to denote the null interval matrix $\begin{bmatrix} \tilde{0} & \tilde{0} & .. \ldots\tilde{0} \\ \tilde{0} & \tilde{0} & \ldots\ldots\tilde{0} \\ . & & \\ . & & \\ \tilde{0} & \tilde{0} & \ldots\ldots\tilde{0} \end{bmatrix}$

2.4 IDENTITY INTERVAL MATRIX

We use I to denote the identity matrix $\begin{bmatrix} I & 0 & .. \ldots 0 \\ 0 & I & \ldots\ldots 0 \\ . & & \\ . & & \\ 0 & 0 & \ldots\ldots I \end{bmatrix}$ and $\tilde{I}$ to denote the identity interval matrix $\begin{bmatrix} \tilde{I} & \tilde{0} & .. \ldots\tilde{0} \\ \tilde{0} & \tilde{I} & \ldots\ldots\tilde{0} \\ . & & \\ . & & \\ \tilde{0} & \tilde{0} & \ldots\ldots\tilde{I} \end{bmatrix}$

2.5 EQUIVALENT INTERVAL MATRIX

If $m(\tilde{A}) = m(\tilde{B})$ then the interval matrices $\tilde{A}$ and $\tilde{B}$ are said to be equivalent and is denoted by $\tilde{A} \approx \tilde{B}$

In particular if $m(\tilde{A}) = m(\tilde{B})$ and $w(\tilde{A}) = w(\tilde{B})$, then $\tilde{A} \approx \tilde{B}$

2.6 ZERO INTERVAL MATRIX

If $m(\widetilde{A}) = 0$, then we say that $\widetilde{A}$ is a zero interval matrix. In particular

if $m(\widetilde{A}) = 0$ and $w(\widetilde{A}) = 0$, then $\underset{\sim}{A} = \begin{bmatrix} [0,0] & [0,0] & \text{.. ...}[0,0] \\ [0,0] & [0,0] & \text{........}[0,0] \\ . & & \\ . & & \\ [0,0] & [0,0] & \text{........}[0,0] \end{bmatrix}$

2.7 NON-ZERO INTERVAL MATRIX

If $m(\widetilde{A}) = 0$ and $w(\widetilde{A}) \neq 0$, then $\underset{\sim}{A} = \begin{bmatrix} [0,0] & [0,0] & \text{.. ...}[0,0] \\ [0,0] & [0,0] & \text{........}[0,0] \\ . & & \\ . & & \\ [0,0] & [0,0] & \text{........}[0,0] \end{bmatrix} \approx \tilde{0}$

$\widetilde{A} \neq \tilde{0}$ (i.e. $\widetilde{A}$ is not equivalent to $\tilde{0}$), then $\widetilde{A}$ is said to be a non-zero interval matr**ix.**

2.8 IDENTITY INTERVAL MATRIX

If $m(\widetilde{A}) = I$, then we say that $\widetilde{A}$ is a identity interval matrix. In particular

If $m(\widetilde{A}) = I$ and $w(\widetilde{A}) = 0$, then $\underset{\sim}{A} = \begin{bmatrix} [1,1] & [0,0] & \text{.. ...}[0,0] \\ [0,0] & [1,1] & \text{........}[0,0] \\ . & & \\ . & & \\ [0,0] & [0,0] & \text{........}[1,1] \end{bmatrix}$ and also

If $m(\widetilde{A}) = 0$ and $w(\widetilde{A}) \neq 0$, then $\widetilde{A} = \begin{bmatrix} [1,1] & [0,0] & \text{.. ...}[0,0] \\ [0,0] & [1,1] & \text{........}[0,0] \\ . & & \\ . & & \\ [0,0] & [0,0] & \text{........}[1,1] \end{bmatrix}$

2.9 THE ARITHMETIC OPERATIONS ON INTERVAL MATRICES

We define the arithmetic operations on interval matrices as follows.

If $\tilde{A},\tilde{B} \in D^{m\times n}$, $\tilde{\alpha} \in D^n$ and $\tilde{\alpha} \in D$

i. $\tilde{\alpha}\tilde{A} = (\tilde{\alpha}\tilde{a}_{ij})_{1\le i\le m,\ 1\le j\le n,}$

ii. $\tilde{A}+\tilde{B} = (\tilde{a}_{ij}+\tilde{b}_{ij})_{1\le i\le m,\ 1\le j\le n,}$

iii. $\tilde{A}-\tilde{B} = \begin{cases} (\tilde{a}_{ij}-\tilde{b}_{ij})_{1\le i\le m,\ 1\le j\le n,} & if \tilde{A}\ne\tilde{B} \\ \tilde{A}-dual(\tilde{A}) = \tilde{0} = 0, if \tilde{A}=\tilde{B} \end{cases}$

iv. $\tilde{A}\tilde{B} = (\sum_{k=1}^{n}\tilde{a}_{ik}\tilde{b}_{kj})_{1\le i\le m,\ 1\le j\le n,}$

v. $\tilde{A}\tilde{X} = (\sum_{j=1}^{n}\tilde{a}_{ij}\ \tilde{X})_{1\le i\le m,}$

3. MAIN RESULTS: EIGEN VALUES OF AN INTERVAL MATRIX

3.1 DETERMINANT OF AN INTERVAL MATRIX

We define the determinant of a square interval as in the case of real square matrix except that the determinant of an interval matrix is an interval number.

That is det $\tilde{A} = |\tilde{A}| = \sum \tilde{a}_{ij}\tilde{A}_{ij}$

Where $\tilde{A}_{ij}$ is the cofactor of $\tilde{a}_{ij}$ with usual meaning.

It is easy to see that most of the properties of determinants of interval matrices are hold good for the determinants of interval matrices under the modified interval arithmetic.

Example:1

Let $\tilde{A} = [A, \bar{A}] = \begin{bmatrix} [1,2] & [3,4] \\ [-9,1] & [8,10] \end{bmatrix}$

Then $|\tilde{A}| = \begin{vmatrix} [1,2] & [3,4] \\ [-9,1] & [8,10] \end{vmatrix}$

$$= [1,2][8,10] - [-9,1][3,4]$$
$$= [8,19] - [-32,4]$$
$$= [4,51]$$

Example:2

Let $\tilde{A} = \begin{bmatrix} [0,2] & [1,3] \\ [3,5] & [5,7] \end{bmatrix}$

Then $|\tilde{A}| = \begin{vmatrix} [0,2] & [1,3] \\ [3,5] & [5,7] \end{vmatrix}$

$= [0,2][5,7] - [1,3][3,5]$

$= [-3,9]$

3.2 CHARACTERISTIC EQUATION

Consider the linear transformation $\tilde{Y} = \tilde{A}\tilde{X}$

In general, this transformation transforms a column vector $\tilde{X} = \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ . \\ . \\ \tilde{x}_n \end{bmatrix}$ in to another column vector

$\tilde{Y} = \begin{bmatrix} \tilde{y}_1 \\ \tilde{y}_2 \\ . \\ . \\ \tilde{y}_n \end{bmatrix}$ by means of the square interval matrix $\tilde{A}$

Where $\tilde{A} = \begin{bmatrix} \tilde{a}_{11} & \tilde{a}_{12} & . & . & \tilde{a}_{1n} \\ \tilde{a}_{21} & \tilde{a}_{22} & . & . & \tilde{a}_{2n} \\ . & & & & \\ . & & & & \\ \tilde{a}_{n1} & \tilde{a}_{n2} & . & . & \tilde{a}_{nn} \end{bmatrix}$

If a vector $\tilde{X}$ is transformed into a scalar multiple of the same vector

(i.e) $\tilde{X}$ is transformed in to $\lambda\tilde{X}$, then $\tilde{Y} = \lambda\tilde{X} = \tilde{A}\tilde{X}$

(i.e) $\lambda\tilde{X} = \tilde{A}\tilde{X} = \lambda\tilde{I}\tilde{X}$ where $\tilde{I}$ is the unit interval matrix of order n

$\tilde{A}\tilde{X} - \lambda\tilde{I}\tilde{X} = \tilde{0}$

$(\tilde{A}-\lambda\tilde{I})\tilde{X}=\tilde{0}$ ……………….(1)

$$\left[\begin{bmatrix}\tilde{a}_{11} & \tilde{a}_{12} & . & . & \tilde{a}_{1n}\\ \tilde{a}_{21} & \tilde{a}_{22} & . & . & \tilde{a}_{2n}\\ . & & & & \\ . & & & & \\ \tilde{a}_{n1} & \tilde{a}_{n2} & . & . & \tilde{a}_{nn}\end{bmatrix}-\lambda\begin{bmatrix}\tilde{I} & \tilde{0} & . & . & \tilde{0}\\ \tilde{0} & \tilde{I} & . & . & \tilde{0}\\ . & & & & \\ . & & & & \\ \tilde{0} & \tilde{0} & . & . & \tilde{I}\end{bmatrix}\right]\begin{bmatrix}\tilde{x}_1\\ \tilde{x}_2\\ .\\ .\\ \tilde{x}_n\end{bmatrix}=\begin{bmatrix}\tilde{0}\\ \tilde{0}\\ .\\ .\\ \tilde{0}\end{bmatrix}$$

$$\begin{bmatrix}\tilde{a}_{11}-\lambda & \tilde{a}_{12} & . & . & \tilde{a}_{1n}\\ \tilde{a}_{21} & \tilde{a}_{22}-\lambda & & . & .\tilde{a}_{2n}\\ . & & & & \\ . & & & & \\ \tilde{a}_{n1} & \tilde{a}_{n2} & . & . & \tilde{a}_{nn}-\lambda\end{bmatrix}\begin{bmatrix}\tilde{x}_1\\ \tilde{x}_2\\ .\\ .\\ \tilde{x}_n\end{bmatrix}=\begin{bmatrix}\tilde{0}\\ \tilde{0}\\ .\\ .\\ \tilde{0}\end{bmatrix}$$

$$\begin{aligned}&(\tilde{a}_{11}-\lambda)\tilde{x}_1+\tilde{a}_{12}\tilde{x}_2+\quad . \quad . \quad +\tilde{a}_{1n}\tilde{x}_n=\tilde{0}\\ &\tilde{a}_{21}\tilde{x}_1+(\tilde{a}_{22}-\lambda)\tilde{x}_2+\quad . \quad .+\tilde{a}_{2n}\tilde{x}_n=\tilde{0}\\ &.\\ &.\\ &\tilde{a}_{n1}\tilde{x}_1+\tilde{a}_{n2}\tilde{x}_2+\quad . \quad .+(\tilde{a}_{nn}-\lambda)\tilde{x}_n=\tilde{0}\end{aligned}$$ ……….(2)

This system of equation will have a non-trivial solution if $\left|\tilde{A}-\lambda\tilde{I}\right|=\tilde{0}$

$$\begin{vmatrix}\tilde{a}_{11}-\lambda & \tilde{a}_{12} & . & . & \tilde{a}_{1n}\\ \tilde{a}_{21} & \tilde{a}_{22}-\lambda & & . & .\tilde{a}_{2n}\\ . & & & & \\ . & & & & \\ \tilde{a}_{n1} & \tilde{a}_{n2} & . & . & \tilde{a}_{nn}-\lambda\end{vmatrix}=\begin{bmatrix}\tilde{0}\\ \tilde{0}\\ .\\ .\\ \tilde{0}\end{bmatrix}=0$$ ………..(3)

The equation $\left|\tilde{A}-\lambda\tilde{I}\right|=0$ or the equation (3) is said to be the characteristic equation of the transform or the characteristic equation of the matrix $\tilde{A}$.

Solving $\left|\tilde{A}-\lambda\tilde{I}\right|=0$, we get n roots for λ, these roots are called the characteristic roots or Eigen values.

Note

If $\tilde{X}_r$ be the non-zero vector satisfying $\tilde{A}\tilde{X}-\lambda\tilde{X}$

When $\tilde{\lambda}=\tilde{\lambda}_r$, $\tilde{X}_r$ is said to be the Eigen vector or latent values of the matrix $\tilde{A}$ corresponding to $\tilde{\lambda}_r$

3.3 Working Rule to find Characteristic equation

Method 1

Let $\tilde{A}=\begin{bmatrix}\tilde{a}_{11} & \tilde{a}_{12} & \tilde{a}_{13}\\ \tilde{a}_{21} & \tilde{a}_{22} & \tilde{a}_{23}\\ \tilde{a}_{31} & \tilde{a}_{32} & \tilde{a}_{33}\end{bmatrix}_{3X3}$ be a 3X3 matrix

Then the characteristic equation is $\left|\tilde{A}-\lambda\tilde{I}\right|=0$

Method 2

2 X 2 Matrixes

Let us assume 2X 2 matrix $\tilde{A}=\begin{bmatrix}\tilde{a}_{11} & \tilde{a}_{12}\\ \tilde{a}_{21} & \tilde{a}_{22}\end{bmatrix}$

If $\tilde{A}$ is a square matrix of order 2, then its characteristic equation can be written as $\lambda^2-\tilde{D}_1\lambda+\tilde{D}_2=0$

Where $\tilde{D}_1$=sum of the main diagonal interval elements= $\tilde{a}_{11}+\tilde{a}_{22}$

$\tilde{D}_2$ =Determinant value of $\tilde{A}=\left|\tilde{A}\right|=\begin{vmatrix}\tilde{a}_{11} & \tilde{a}_{12}\\ \tilde{a}_{21} & \tilde{a}_{22}\end{vmatrix}$

3 X 3 Matrix

Let $\tilde{A} = \begin{bmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \tilde{a}_{13} \\ \tilde{a}_{21} & \tilde{a}_{22} & \tilde{a}_{23} \\ \tilde{a}_{31} & \tilde{a}_{32} & \tilde{a}_{33} \end{bmatrix}$ be a 3X3 matrix, and then its characteristic equation can be written as

$$\lambda^3 - \tilde{D}_1\lambda^2 + \tilde{D}_2\lambda - \tilde{D}_3 = 0$$

Where $\tilde{D}_1$ =sum of the main diagonal interval elements= $\tilde{a}_{11} + \tilde{a}_{22} + \tilde{a}_{33}$

$\tilde{D}_2$ =sum of the minors diagonal interval elements

$$= \begin{vmatrix} \tilde{a}_{22} & \tilde{a}_{23} \\ \tilde{a}_{32} & \tilde{a}_{33} \end{vmatrix} + \begin{vmatrix} \tilde{a}_{11} & \tilde{a}_{13} \\ \tilde{a}_{31} & \tilde{a}_{33} \end{vmatrix} + \begin{vmatrix} \tilde{a}_{11} & \tilde{a}_{21} \\ \tilde{a}_{21} & \tilde{a}_{22} \end{vmatrix}$$

$\tilde{D}_3$ =Determinant value of $\tilde{A}$

$$|\tilde{A}| = \begin{vmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \tilde{a}_{13} \\ \tilde{a}_{21} & \tilde{a}_{22} & \tilde{a}_{23} \\ \tilde{a}_{31} & \tilde{a}_{32} & \tilde{a}_{33} \end{vmatrix}$$

Example to find the characteristic equation

Let us consider the interval matrix $\begin{bmatrix} [1,2] & [3,4] \\ [-9,1] & [8,10] \end{bmatrix}$

The characteristic equation of the interval matrix $|\tilde{A} - \lambda\tilde{I}| = 0$

$$|\tilde{A}| = \begin{vmatrix} [1,2] & [3,4] \\ [-9,1] & [8,10] \end{vmatrix}$$

$$= [1,2][8,10] - [-9,1][3,4]$$

$$= [8,19] - [-32,4] = [4,51]$$

The characteristic equation of the interval matrix $\left|\tilde{A}-\lambda\tilde{I}\right|$

$$= \lambda^2 - \left[\frac{137}{14}, \frac{157}{14}\right]\lambda + [4,51]$$
$$= \tilde{0} = 0$$

Eigen values in interval matrix.

Let $\tilde{A} = \left[\tilde{a}_{ij}\right]$ be a square matrix.

The characteristic equation of the interval matrix $\tilde{A}$ is $\left|\tilde{A}-\lambda\tilde{I}\right| = \tilde{0} = 0$.

The roots of the characteristic equation are called Eigen values of $\tilde{A}$

Example to find the Eigen values of an interval matrix

Let us consider the interval matrix $\tilde{A}$ is

$$\begin{bmatrix} [0,2] & [1,3] \\ [3,5] & [5,7] \end{bmatrix}$$

Using power method, to get in Eigen values $\tilde{x}_1 = \tilde{A}\tilde{x}_0 = \lambda\,\tilde{x}_1^1$

$$\tilde{x}_2 = \tilde{A}\tilde{x}_1^1 = \lambda\,\tilde{x}_2^{\,1} \quad \tilde{x}_3 = \tilde{A}\tilde{x}_2^{\,1} = \lambda\,\tilde{x}_3^{\,1}$$

Similarly, we can find out the corresponding Eigen values and Eigen vectors.

$$\begin{bmatrix} [0,2] & [1,3] \\ [3,5] & [5,7] \end{bmatrix}\begin{bmatrix} [1,1] \\ [0,0] \end{bmatrix} = [3,5]\begin{bmatrix} \left[0,\frac{1}{2}\right] \\ [1,1] \end{bmatrix} \begin{bmatrix} [0,2] & [1,3] \\ [3,5] & [5,7] \end{bmatrix}\begin{bmatrix} \left[0,\frac{1}{2}\right] \\ [1,1] \end{bmatrix} = [5,9] \begin{bmatrix} \left[\frac{1}{6},\frac{10}{21}\right] \\ [1,1] \end{bmatrix}$$ in this way,

we can find out the corresponding Eigen values $[-4,5]$

4. Conclusion

In this work, Power methods have been successfully applied to interval matrix; find the solution of the Eigen values and Eigen vectors. This method can be concluded that the method is very powerful and efficient techniques in finding exact solutions. It is to be noted that the solution set (Eigen values and Eigen vectors) obtained by this method is sharper than the other techniques.

ACKNOWLEDGEMENT

First and foremost I owe my hearted thanks to god who has bestowed his blessings on me to complete the work successfully.

With the deep sense of gratitude, I express my profound thanks to my guide Dr. E.C.Henry Amirtharaj, Ph.D, Reader in Mathematics, Bishop Heber College, Trichy Who suggested me the topic and helped me a lot in completing the dissertation; also I am deeply and extremely grateful to her excellent guidance and constant encouragement. I also express my sincere thanks to Mrs.K.Thamilselvi, H.O.D, Dept.of Mathematics, A.M.S Engineering College, Namakkal, for her encouragement.

At the more personal level, I wish to express my sincere thanks to all of my friends and mainly my parents, especially to K.Pushparaj & Vimala, Dept.of Mathematics, A.M.S Engineering College, Namakkal for their kind encouragement and support.

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